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THEORY OF OPTIMUM STRUCTURES: AN ANNOTATED BIBLIOGRAPHY

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SB-62-49

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Compiled by
G. R. EVANS and R. A. EISENTRAUT

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Lockheed

MISSILES & SPACE COMPANY
A GROUP DIVISION OF LOCKHEED AIRCRAFT CORPORATION
SUNNYVALE, CALIFORNIA

ABSTRACT

A general problem in the theory of structures is to determine the configuration which has minimum weight from all those which can support the prescribed loads. Some progress has been made in the solution of this problem, and it was the purpose of the search to determine the present state of the theory. The literature contains great amounts of material of a less general nature on minimum weight design of specific structures, and the references cited for these are incidental and not complete. The major effort was to find references that contributed to the solution of the general problem stated above.

Arrangement is by broad subject areas and chronologically within each subject category. The period covered is from 1950 to September 1962. No attempt was made to search the literature prior to 1950 because of the dearth of published information, applicable to the problem, issued between 1904-1950 (1904 was the year in which Michell published his criterion for the minimum weight of trusses).

Search completed September 1962.

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INTRODUCTION

The theory of structures is primarily concerned with the analysis of given configuration, and comparatively little effort has gone into the question of optimization. The general problem is to determine the configuration which has minimum weight, and which will support the prescribed loads and will be contained within a prescribed space. Some progress has been made in the solution of this problem, and it was the purpose of the literature search to determine the present state of the theory.* The literature is relatively abundant with solutions to less general problems such as minimum weight design of columns, panels, beams, frames, plates, shells, etc., and the references cited for these are incidental and not complete (Books such as those written by Shanley, Citation #47, or Gerard, Citation #21, contain large numbers of references). The major effort has been spent in obtaining material which contributes to the solution of the general problem stated above.

The first significant contribution to the general problem was made by Michell in 1904, who derived a sufficient condition for a truss to be of minimum volume. His work was an extension of a theorem given by Maxwell before the turn of the century. No significant progress was made in the solution of the general problem until 1957, when Drucker and Shield applied the theorems of limit analysis to obtain new criteria required for a minimum. These methods have been extended since then, but the criteria are still of such a restrictive nature that optima can only be found in certain special cases. Much work remains to be done toward the solution of the general question.

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* The contents of the reference cited regarding the solution of the general problem have been summarized and discussed in Lockheed Missiles & Space Company, Theory of Optimum Structure, by R. A. Eisentraut, Report 8-40-62-6, Sunnyvale, California, October 1962.

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Part I

OPTIMUM OR MINIMUM-WEIGHT DESIGN OF SPECIFIC STRUCTURES

A. COLUMNS IN COMPRESSION

1. Kenedi, R.M., Smith, W.S. and Fahmy, F.O.
Light structures – Research and its application to economic design. TRANS. INSTN. ENGRS. & SHIPBLDERS IN SCOTLAND v. 99, pt. 4, p. 207-52, (Discussion) p. 253-64, 1955-56. (With Bibliography)

A presentation of the experimental behavior of thin-walled structs and beams; charts presenting design information are included.

2. Kirste, L.

Minimum weight compression rods. OST. ING-ARCH. v. 12, n. p/2, p. 36-41, Nov 1958. (In German)

Introduces a modified version of Johnson's formula for short columns by postulating that for zero slenderness ratio the column load capacity for overall buckling be equal to the load capacity for local crippling. This is an analytically convenient assumption; yet it is to be seen how realistic it is for a given specific combination of material and cross section. No supporting evidence is given. On the basis thus established, minimum-weight design criteria are developed for thin-walled round, hexagonal and octagonal tubular compression members.

3. Kirste, L.
Consideration of the problem of minimum weight.
Z. FLUGWISS. v. 12, p. 352-358, Dec 1960.
(In German)

The minimum-weight design of "one-dimensional" elements of construction is considered. Interference between slimness in buckling and local failure leads to an optimum for the "spreading" of the cross section. To make the "useful fatigue" as high as possible, it

3. (cont'd) should occur in the plastic range. Efficiency increases with the "loading density" P/σ_o, i.e. with the ratio of applied effort to pure compression strength or yield limit and to the square of the length of reference. Among simple sections the round tube is best; polygonal tubes are better, the higher the number of faces. Eventually, they must arrive at the same values. With small over-all loading densities, the plain-wall tubes have to be developed into frameworks to increase the local loading densities.

B. FRAMES

4. Heyman, J.
Plastic design of plane frames for minimum weight. STRUCT. ENG. v. 31, p. 125-129, 1953.

The principles of limit analysis are applied in the minimum weight design of rigid frames with constant cross-section between joints.

5. Foulkes, J.
Minimum weight design of structural frames. PROC. ROY. SOC. (LONDON) Series A, v. 223, p. 482-494, 1954.

This paper examines the problem of assigning economical sections which are constant between joints to frames which derive their strength from a bending action. The theory of plastic collapse is used to determine the strength of a design, and a virtual work approach is used to derive the equations governing the problem. These equations show that a linear function whose variables are constrained by linear inequalities has to be minimized. In order to illustrate the meaning of these equations a geometrical analogue is described and this analogue is then used to prove a few general theorems. The paper concludes with a demonstration of the way in which these theorems can be applied to the design of a simple practical structure.

6. Prager, W.
Minimum-weight design of portal frame. PROC. AM. SOC. CIVIL ENG. v. 82 (J. ENG. MECH. DIV.) n. EM 4, Paper no. 1073, Oct 1956, 10p.

7. Heyman, J.
Minimum weight of frames under shakedown loading. PROC. AM. SOC. CIV. ENG. v. 82 (J. ENG. MECH. DIV.) n. EM4, Paper n. 1790, Oct 1958, 25p.

Minimum weight design of framed structures under both fixed and independent by varying loads is discussed. Numerical example for two span beam, for which complete solution is obtained, and results corresponding to two types of loading are compared; slightly inexact iterative method for minimum weight design of frame of any degree of complexity.

8. Heyman, J.
On the absolute minimum weight design of framed structures. QUART. J. MECH. AND APPL. MATH. v. 12, p. 314-324, 1959.

If the cross-section of the members of a frame is varied continuously so that the 'strength' of the frame follows the bending moment diagram, then, provided the design is carried out a certain way, the frame will have absolute minimum material consumption. This theorem is proved separately here, although for plastic design the theorem is a special case of the general principles given by Drucker and Shield. However, this special theorem applies equally well to certain completely elastic structures. The theorem is applied to determine the minimum weight of some simple continuous beam systems, and a trial and error 'relaxation' technique is developed. These designs are compared with more practical designs in which the cross-section of the members varies discontinuously.

9. Livesley, R.K.
Optimum design of structural frames for alternative systems of loading. CIV. ENG. (LONDON) v. 54, n. 636, p. 737-740, June 1959.

A clear graphical representation of the problem of least-weight design of rigid jointed frames under alternate loading conditions, using theory of plastic collapse. Load system may vary between prescribed limits, and is represented (for the two-load system considered) by a "load diagram". Some possible failure mechanisms are represented on a "design diagram." For each mechanism a limiting value is found for which design is safe (i.e. on point of collapse) under all load combinations. Assuming weight to be proportional to the fully plastic moment, lines of constant

9. (cont'd) weight are drawn on the "design diagram." The solution is the line that touches the safe region defined by all possible mechanisms. Extension to more loading systems and more plastic moments can be made by linear programming, requiring use of a digital computer. Approach is useful in obtaining a general picture of structural behavior. Minimum weight design is one that most successfully balances effect of a number of extreme loading cases.

10. Heyman, J.
On the minimum-weight design of a simple
portal frame. INTER. J. MECH. SCI. v. 1,
p. 121-134, Jan 1960.

An extention of the analysis of Foulkes which employs a plastic collapse theory for minimum-weight design, to (a) frames with unequal columns and (b) frames with tapered members. Results for the latter are called "absolute minimum-weight design," and show a large weight saving over prismatic-member frames in three simple examples. A practical method of approximating absolute minimum-weight design is also given.

C. PLATES IN BENDING

11. Hopkins, H.G. and Prager, W.
Limits of economy of materials in plates.
TRANS. ASME J. APPL. MECH. v. 22, n. 3,
p. 372-374, Sep 1955.

The minimum weight design of a simply supported circular plate under a uniform transverse load is investigated using an intuitive concept of the behavior of minimum weight structures. The plate material is assumed to be plastic-rigid and obey Tresca's yield condition and the associated flow rule. The criterion of failure is that of limit analysis. Stepped plates are considered, and the plate of continuously varying thickness is treated as the limiting case when the number of steps go to infinity.

12. Prager, W.
Minimum weight design of plates.
DE INGENIEUR Sect. 0, v. 67, p. 0.141-
0.142, Dec 1955.

The minimum weight design of plates in bending is investigated using an intuitive concept of the behavior of minimum weight structures.

13. Freiberger, W. and Teklinalp, B.
Minimum weight design of circular plates.
J. MECH. AND PHYS. SOLIDS v. 4, p. 294-299,
1956.

The minimum weight design of circular symmetrically loaded solid and sandwich plates in bending was obtained directly using the calculus of variations. The plate is assumed to obey the von Mises yield condition, and it is shown that the minimum weight design admits a failure mechanism for which the mechanical energy dissipated per unit volume has a constant value throughout.

14. Onat, E. T., Schumann, W. and Shield, R. T.
Design of circular plates for minimum weight.
J. APPL. MATH. PHYS. (ZAMP) v. 8, p. 485-499,
1957.

The problem considered here is the minimum weight design of a circular plate of variable thickness subjected to rotationally symmetric loads and edge support. The direct design procedures developed by Drucker and Shield from the theorems of limit analysis are used. The designs obtained are absolute minimum weight designs for the sandwich plate but the minimum is a relative one for the solid or homogeneous plate.

15. Prager, W. and Shield, R. T.
MINIMUM WEIGHT DESIGN OF CIRCULAR PLATES UNDER ARBITRARY LOADING.
Brown Univ. Tech. Rept. DA-4564/4, 1958.
(Also in: J. APPL. MATH. PHYS. (ZAMP)
v. 10, p. 421-426, 1959)

Previous work using the direct design procedures developed by Drucker and Shield to obtain the minimum weight design of circular plates under rotationally symmetric pressure distribution are extended to include arbitrary distributions of pressure over the plates. The required thickness variations are obtained for the case of the sandwich plate for various edge conditions using the Tresca yield condition.

16. Shield, R. T.
Plate design for minimum weight. QUAR. APPL. MATH. v. 18, p. 131-144, 1960.

A discussion of the equations determining the minimum weight design of transversely loaded sandwich plates of arbitrary shape is given. The basic equations are formulated and the four different types of solution are discussed. An inverse method is developed to obtain the minimum volume design for plates with built-in edge conditions. The method is used to obtain the design for an elliptical plate.

17. Eason, G.
The minimum weight design of circular sandwich plates. J. APPL. MATH. PHYS. (ZAMP) v. 11, n. 5, p. 368-375, Sep 1960. (In English)

The direct design methods of Drucker and Shield are used here to investigate the minimum weight design of a circular sandwich plate uniformly loaded over an axially symmetric area when the von Mises yield condition and its associated flow rule are assumed. Comparison is made with earlier results derived for the Tresca yield condition, and are very similar. Thus, the simpler results of the Tresca yield conditions are likely to be adequate in a practical design.

D. PANELS AND PLATES IN COMPRESSION

18. Schalin, P.H.B.
DETERMINATION OF OPTIMUM DIMENSIONS FOR AN AIRCRAFT FUSELAGE SHELL CONSIDERING SPECIAL STIFFNESS CRITERIA AND MINIMUM WEIGHT. SAAB Aircraft Co., Linkoping. Rept. no. TN 8, 1952, 19p.

A consideration of the optimum distribution of material in a fuselage to give minimum weight satisfying given conditions of deflection and slope at the tail. Distribution of radius of gyration is assumed, then methods of calculus of variation are used to find minimum area distribution. A numerical example is given, and conclusion is that, where stiffness is the design criterion, this method gives noticeable weight-saving over scaling-up of area distribution given by strength requirements.

19. Catchpole, E. J.
The optimum design of compression surfaces having unflanged integral stiffeners. J. ROY. AERONAUT. SOC. v. 58, p. 527, 765-768, Nov 1954.

A method is developed enabling rapid determination of the optimum cross-sectional dimensions of compression surfaces having unflanged integral stiffeners, and consideration is given to the effects of practical limitations on the design. The theoretical efficiency of the optimum design is found to be only 85% of that of optimum Z-stringer design.

20. Klein, B.
Certain aspects of the problem of efficient structural design. J. FRANKLIN INST. v. 260, n. 2, p. 107-114, Aug 1955.

The application of calculus of variations for deriving an optimum design is illustrated by an example.

21. Gerard, G.
MINIMUM WEIGHT ANALYSIS OF COMPRESSION STRUCTURES. New York, New York University Press, 1956, 194p.

Most of the works on optimum design of aircraft have been published since the end of World War II. This book is a successful attempt at making a cohesive presentation of the various studies on minimum weight analysis of aircraft structures subject to compression buckling. The book is divided into 11 chapters. Chap. 1 explains the principle of minimum weight design and illustrates how to combine the given intensity of loading and the prescribed leading dimensions of the structure for a "structural index" to be used as a governing parameter for optimum design. Chapter 2 discusses the behavior of composite structures and a general discussion on the optimum design parameters in conjunction with the bending of wing, tail, and fuselage elements which consist of compression covers and supporting structures. Chapters 3-7 analyze the minimum weight configuration of the following structural arrangements: (a) Stringer panel-rib construction; (b) transversely stiffened plates; (c) post construction; (d) multicell construction; (e) post-stringer construction; (f) sandwich plate construction; (g) sandwich-box construction; (h) unstiffened cylindrical shells; (i) stringer panel-frame shell; (j) sandwich shell. Chapter 8 compares the various types of construction, using 75S-T aluminum alloy as the typical material. The ranges of efficient structural application of each form of construction can be established with

21. (cont'd) design charts which are included. A discussion of the possible deviation of the previous conclusions from practical considerations is presented in Chapter 9. The relative material efficiencies at normal and elevated temperature is given in Chapter 10. A discussion of minimum weight design of structures for which the leading dimensions may be regarded as open dimensions is taken up in the last chapter.

22. Gerard, G. and Becker, H.
HANDBOOK OF STRUCTURAL STABILITY. PART VII. STRENGTH OF THIN-WING CONSTRUCTION. National Aeronautics and Space Administration, Washington, D.C. Rept. no. NASA TN D-162, Sep 1959. (In cooperation with New York Univ. Coll. of Engineering, N.Y.) ASTIA AD-225 875.

The stability of various forms of stiffened and sandwich plates which have been considered for the compression covers of thin wings is presented in terms of orthotropic plate theory. Design charts are presented and methods of evaluating the elastic constants associated with each type of plate are reviewed. Buckling and failure of multiweb, multipost, and multipost-stiffened forms of beam construction are considered in terms of available theoretical and experimental results. The pertinent findings of minimum-weight analyses are presented throughout the report as an aid in design.

23. Yusuff, S.
Design for minimum weight – considerations based on the long wave instability of stiffened plates in compression. AIRCR. ENG. v. 32, p. 380, 228-294, Oct 1960.

A continuation of an analysis presented by the author in August 1958 issue of Journal of Aero/Space Sci. The present problem is that of minimum weight design of stiffened compression panels having either Z-section or integrally machined unflanged stiffeners. The design criterion is based primarily on a plate instability mode in which the plate and stiffeners buckle simultaneously over a length equal to the pin-ended length of the panel, after being subjected to equal stress. Other types of instability are discussed but are not considered critical for the types of panels commonly used in aircraft construction. Graphs and tables are presented.

24. Davidson, J.R. and Dalby, J.F.
OPTIMUM DESIGN OF INSULATED COMPRESSION PLATES SUBJECTED TO AERODYNAMIC HEATING. National Aeronautics and Space Administration. Rept. no. NASA TN D-520, Jan 1961, 53p.

A method to determine the minimum weight design of insulation and load-carrying structure has been applied to insulated stressed-skin compression structures subjected to aerodynamic heating. Charts have been prepared for four materials and for the loading parameters associated with compressive yield, buckling, and postbuckling failure. Additional charts are presented for conditions where more than one failure mode governs.

E. BEAMS IN BENDING

25. Vargo, L.G.
Nonlinear minimum-weight design of planar structures. J. AERO. SCI. v. 23, n. 10, p. 956-960, Oct 1956.

The development of a direct method for determining the minimum-weight fully plastic moments of a planar structure (such as a beam, bent on truss) is presented. The method allows use of a general type of nonlinear relation between the fully plastic moments and the areas of the series of cross sections used. Solutions for a two-span beam and a rectangular beam bent under concentrated loads are obtained, and are compared with solutions based on a linear relation between those quantities. The nonlinear relation is found to result in a weight saving of as much as 15% for the beam problem.

26. Krishnan, S. and Shetty, K.V.
A method of minimum weight design for thin-walled beams. STRUCT. ENGR. v. 39, n. 5, p. 174-180, May 1961.

A determination of optimum dimensions for thin-walled channel, box, and I-sections, unstiffened or stiffened by uniformly spaced equal stiffeners, subjected to bending. The formulas for critical stresses of elastic plates are used at yield stress, or without any limitation at all.

F. TRUSSES (See Also Part II, Citation No. 39)

27. Walling, J. L.
LEAST-WEIGHT PROPORTIONS OF BRIDGE TRUSSES. Univ. Ill. Engng. Exp. Sta. Bull. no. 417, 1953, 56p.

The determination of least-weight proportions of bridge trusses lends itself to mathematical investigation. The procedure here presented consists of the adaption of the theory of maxima and minima to solving for the proportions of a truss outline, such that the volume of metal in the truss is a minimum. Explanations are given of the assumptions and approximations upon which the calculations are based and concerning the resolution of complications interspersed by modern design specifications. Calculations were performed to determine least-weight proportions and theoretical least weights of simple span, through-type, double-track, open-timber-deck railway bridges having 68 different combinations of truss type, panel length, span length, and live load. In general, the results of these calculations show that weight savings can be accomplished by designing these trusses somewhat deeper than is normally done by present-day designers.

28. Sved, G.
The minimum weight of certain redundant structures. AUSTRAL. J. APPL. SCI. v. 5, n. 1, p. 1-9, Mar 1954.

Variation of weight of a plane, pin-jointed structure of n bars involving k redundancies is investigated under a fixed load system. Assuming any constant ratio between allowable tensile and compressive unit stresses, least weight will be attained when any combination of $n-k$ bars is arranged so as to form a determinate structure.

29. Barta, J.
On the minimum weight of certain redundant structures. ACTA TECH. ACAD. SCI. HUNGAR. v. 18, p. 67-76, 1957. (German, French and Russian summaries)

For statically indeterminate structures, the minimum weight design occurs when sufficient constaints (i.e., redundancies) are removed so that structure becomes statically determinate. The resulting design may not be unique, but the weight is an absolute minimum. For example, for a redundant truss, sufficient bars are removed to make

29. (cont'd) the truss statically determinate, but there may be several ways of doing this, leading to different designs all having the same (minimum) weight. The principle has been known for some time, but does not appear to have been discussed in the literature; Sved gave a restricted proof for an idealized stress-strain curve (Austral. J. Appl. Sci. v. 5, p. 109, 1954), but the present proof is valid for any (reasonable) stress-strain law.

30. Poocza, A.
Minimum weight of high tension towers loaded in torsion. BAUTECHNIK v. 35, n. 10, p. 399-400, Oct 1958. (In German)

The structure considered is a prismatic space truss of rectangular cross section; the four lateral faces of the structure are simple plane trusses. The extent of the structure is supposed to be limited in such a way that the cross section shall not exceed a circle of a fixed diameter. Design stresses of tension and compression bars are assumed to be equal. Taking into consideration no other load than torsion, the structure of minimum weight is found to be composed of cubic cells, i.e., the cross section of the structure is quadratic and all diagonal bars of the lateral faces form angles of 45 degrees with the vertical bars.

31. Laushey, L.M.
Direct design of optimum indeterminate trusses. PROC. AMER. SOC. CIV. ENGRS. v. 84, ST 8 (J. Struct. Div.), Paper 1867, Dec 1958, 35p.

A method is proposed for the direct design of indeterminate trusses. The principle of potential work is introduced to obtain the maximum compatible stresses for the bars. Redundants are selected to yield the minimum weight of truss. The final bar areas follow directly by dividing the forces in static equilibrium by the stresses satisfying continuity. The relative weights of alternative structures and the optimum structure are revealed by the direct-design method.

32. Schmidt, L.C.
Fully-stressed design of elastic redundant trusses under alternative load systems. AUSTRAL. J. APPL. SCI. v. 9, n. 4, p. 337-348, Dec 1958.

32. (cont'd) Value of paper lies in extension to trusses subject to more than one load system. An assumption is made that minimum weight is attained when each member is fully stressed under at least one load system. Two iteration methods for obtaining such a solution is proposed, the first is to be used until it is known which load system fully stresses each member, turning then to the rapidly converging second method. Paper includes illustrative examples. Methods are described only for the cases when the same unit stress governs design of all members. But there is no difficulty in extending it to cover unequal allowable stresses in tension and compression.

G. PRESSURE SURFACES

33. Hoffman, G.A.
MINIMUM-WEIGHT PROPORTIONS OF
PRESSURE VESSEL HEADS. Rand Corporation,
Aero-Astronautics Dept., Santa Monica, Calif.
Paper no. P-2137, Nov 1960, 29p.

This paper derives the minimum-weight shapes of head closures for cylindrical pressure vessels. Head configurations, weights and membrane stresses are obtained for thin-shell heads of isotropic materials with uniform internal pressure by restricting the knuckle shear stresses to constant values where possible and by minimizing the weight of prescribed-shape heads. Torispherical, ellipsoidal, and Biezeno-type heads with constant and varying thickness (neglecting the effects of bending) are investigated. Enclosed-volume constancy and supporting structures are also considered. Certain highly efficient shapes are obtained that weigh up to 11% less than the corresponding hemispherical capping closure.

34. Saelman, B.
A note on the minimum-weight design of
spherical and cylindrical pressure vessels.
J. AEROSPACE SCI. v. 28, n. 1, p. 72-73
(Readers' Forum), Jan 1961.

H. SHELLS

35. Freiberger, W.
Minimum weight design of cylindrical shells.
J. APPL. MECH. v. 23, p. 576-580, 1956.

35. (cont'd) The development of a method for designing the variable wall thickness of a cylindrical shell under axial loading and arbitrary pressures to give maximum economy of material is presented. The shell is composed of an ideally plastic material and it is assumed that for maximum economy of material the generic point of the stress state of the shell must move on the edges separating the various smooth domains of the yield surface defined in terms of generalized stresses. Using this assumption and the equations of equilibrium the optimum thickness distribution is obtained for a variety of loading conditions. The criterion of minimum weight design used has its shortcomings: For instance, it is not applicable to entirely smooth yield surfaces.

36. Freiberger, W.
On the minimum weight design problem for cylindrical sandwich shells. J. AERO. SCI. v. 4, p. 847-848, 1957.

The minimum weight design of a symmetrically loaded circular cylindrical sandwich shell with zero body force is derived using the calculus of variations. It is shown that the minimum weight design admits a failure mechanism for which the energy dissipation rate per unit volume has a constant value throughout.

37. Shield, R.T.
On the optimum design of shells. TRANS. ASME, Series E, v. 27, p. 316-322, 1960.

A procedure is developed for obtaining the design of an elastic, perfectly plastic shell or structure which will support prescribed loads and which is the optimum design for a given criterion. The action of body forces is included in the analysis. Some problems in the minimum volume design of a circular cylindrical sandwich shell are solved to illustrate the method, and it is found that only for relatively short shells does the minimum volume design effect an appreciable saving over the membrane design.

I. EXPERIMENTAL METHODS

38. Meyer, J.H.
Test development of structures designed understrength. AERO. ENG. REV. v. 13, n. 10, p. 54-64, Oct 1954.

Stretch testing, in which structures are designed understrength and brought up to strength by successive static tests, is discussed. The benefits of weight saving in general are brought out and an approximate criterion for the value of weight presented.

38. (cont'd) Data are presented on the scatter and stretch observed during various test programs. The sources of inaccuracy of analytical strength determinations are enumerated. Examples of stretch programs are described and the benefits discussed. A design process for exploiting stretch testing is described, and several factors entering into the choice of stretch-test methods are discussed. It is concluded that stretch testing is an extremely valuable tool in developing efficient aircraft structures.

Part II

GENERAL CONSIDERATION ON OPTIMUM OR MINIMUM-WEIGHT DESIGN

39. Michell, A.G.M.
The limits of economy of material in frame structures. PHIL. MAG. (LONDON), Series 6, v. 8, n. 47, Nov 1904.

A criterion for the minimum weight of trusses is derived from a theorem first given by Maxwell. A truss which satisfies the condition that the strain of all tension members is e and that of all compression members in -e, and no other linear element has a strain greater than e, has minimum volume. In addition, all tension members must be stressed to a prescribed limit and all compression members must be stressed to a possibly different prescribed limit. This criterion can only be satisfied in relatively few cases, some of which are given in this paper.

40. Foulkes, J.
Minimum weight design and the theory of plastic collapse. QUART. APPL. MATH. v. 10, n. 4, p. 347-358, Jan 1953.

An examination of the problem of assigning economical sections to the members of a structure whose geometrical form is given. The criterion of failure is taken to be that of the plastic theory of collapse, and the criterion of minimum weight is employed to determine the best design. A geometrical analog of the equations involved is used to clarify their significance, and such proofs as there are in the text are cast into geometrical terms. A method of solution is suggested at the end of the paper, but the primary concerns of the paper are the general features of the problem.

41. Drucker, D.C. and Shield, R.T.
Design for minimum weight. In INTERNATIONAL CONGRESS FOR APPLIED MECHANICS, 9th, 1956. PROCEEDINGS v. 5, p. 212-222, 1957.

41. (cont'd) Design rather than analysis poses the real problem in machines and structures. The basic for an optimum design is, of course, a compromise between material and fabrication costs within some space limitation. It is always of interest, therefore, to know the minimum weight possible so that an appropriate design may be selected. A very simple formal approach to this problem is outlined for a perfectly plastic material with a completely general yield criterion and then specialized to sheets, membranes and plates. Some of the specific results have been established previously by more elaborate techniques, other appear to be new. Much more work must be done because in all cases the conditions found are sufficient rather than necessary and the question of existence of solutions arises. Also, in applications involving bending of homogeneous beams and plates the minimum is a relative one and not an absolute one as for space structures, sheets, and membranes.

42. Drucker, D.C. and Shield, R.T.
Bounds on minimum weight design. QUART.
APPL. MATH. v. 15, p. 269-281, 1957.

A somewhat limited design procedure for elastic-perfectly plastic structures was developed previously by the authors. ("Design for Minimum Weight", Proc. of the 9th Int. Congress of Applied Mechanics, Brussels, 1956. See previous citation) It is extended here to provide upper and lower bounds on the minimum weight of three dimensional structures in which either direct stresses or bending stresses are negligible. The generalization also includes the influence of body forces. In principle, therefore, such troublesome factors as the weight of the structure itself or centrifugal "forces" may be designed for in a direct manner. Radially symmetric plane stress and plate bending examples are solved to demonstrate direct design procedures.

43. Hemp, E.D.
NOTES ON THE PROBLEM OF THE OPTIMUM
DESIGN OF STRUCTURES. (Great Britain)
College of Aeronautics, Cranfield. Note no. 73,
Jan 1958, 8p.

The urgent need for a systematic approach to the problem of the optimum design of structures is stressed and ideal formulations of these problems are considered. Differential equations and a variational principle are derived for the case of plates loaded in their own planes; these can form the basis for approximate solutions, in the form of optimum distributions of plate thickness and the corresponding stress distributions which are required to equilibrate given systems of external loads.

44. Cox, H. L.
The application of the theory of stability in structural design. J. ROY. AERO. SOC. v. 62, n. 571, p. 497-519, July 1958.

Presents a running account of certain minimum weight approaches to the design of structures subject to instability. Lightly treated arc multiweb beams and multiweb-stringer construction, frame-stringer and honeycomb sandwich construction. A more complete discussion follows on the interaction between primary instability modes of bending and torsion and between local instability and torsion. Finally, brief conclusions are drawn as to the comparative efficiencies of the various forms of construction considered in terms of a pertinent loading index.

45. Hemp, W.S.
THEORY OF STRUCTURAL DESIGN. Advisory Group for Aeronautical Research and Development, Paris, France. Rept. no. 214, Oct 1958. Presented at the Eighth Meeting of the Structures and Materials Panel held 20-25 October 1958, Copenhagen, Denmark. ASTIA AD-233 121. (Also In College of Aeronautics, Cranfield, Great Britain, Rept. no. 115, Aug 1958, 63p.)

The theory of structures is chiefly concerned with the calculation of stresses in a given structure under given external conditions of loading and temperature. The real problem of structural design, however, in aeronautics especially, is to find that structure which will equilibrate the external loads without failure or undue deformation, under such conditions of temperature as may be appropriate, and which at the same time will have the least possible weight. The solution of this general design problem is obviously very difficult, and cannot be resolved at the present time. However, on the basis of certain classical theorems due to Maxwell and Michell, and using methods and suggestions derived from these theorems by H. L. Cox, certain progress can be made and, in addition, point the way to profitable lines of research. Classical results and their current application are reviewed, mathematical theory for the 2-dimensional case is developed, and a number of special solutions is derived.

46. Dickinson, J.R.
Computer program for system optimization.
TRANS. ENG. INST. CANADA v. 2, n. 4,
p. 157-161, Dec 1958.

There is a possibility of programming the computer to optimize design of complex systems or structures by method which adjusts set of independent design variables to minimize some important criterion such as cost of system or weight of structure: program optimizes nonlinear functions of up to six independent variables and can restrain these variables within upper and lower limits: method is refined gradient technique using efficient extrapolation technique to accelerate process of solution.

47. Shanley, F.R.
WEIGHT-STRENGTH ANALYSIS OF
AIRCRAFT STRUCTURES. 2nd Edition.
New York, Dover Publications, Inc.
1960, 404p.

This book is an unabridged republication of the first edition published in 1952 (McGraw-Hill, New York), to which have been added separate bibliographies on Optimum Design of Structures and Creep Buckling.

48. Shield, R.T.
OPTIMUM DESIGN METHODS FOR
STRUCTURES. Brown Univ., Div. of
Applied Mathematics, Providence, R.I.
Technical rept. no. 1, Apr 1960, 22p.
(Also In PROC. OF 2ND SYMPOSIUM
ON NAVAL STRUCTURAL MECHANICS.
Brown Univ., Providence, R.I., 5-7 Apr 1960)
(Contract DA 19-020-ORD-5124, Proj. TB2-0001)
ASTIA AD-237 534.

Methods for the optimum design of structures composed of perfectly plastic materials are reviewed. The methods are extended to other materials and a brief discussion of the limitations of the methods is given.

49. Mroz, Z.
ON A PROBLEM OF MINIMUM WEIGHT DESIGN. Brown Univ., Div. of Applied Mathematics, Providence, R.I. Rept. no. C11-59; Technical rept. no. 59, May 1960, 18p. (In cooperation with Warsaw Institute of Basic Technical Research) (Contract Nonr-56210) ASTIA AD-236 729.

The problem of optimal design for perfectly plastic, isotropic structures is analyzed. It is shown that for such structures as plates or shells, an extremum of the volume, if it exists, may be either a local maximum or a minimum.

50. Hilton, H.H. and Feigen, M.
Minimum weight analysis based on structural reliability. J. AEROSPACE SCI. v. 27, n. 9, p. 641-652, Sep 1960.

Deals with systems in which failure of each element is independent of that of all other elements for any given condition of loading, and failure of any single element brings about failure of the entire structure. In practice such structures are not often met except as much simplified idealization. Assuming that the probability of failure of the system is small, this leads to the conclusion that it is nearly equal to the sum of the probabilities of failure of all individual members. Shows that, contrary to a common statement, optimum design requires that the various elements in a given system have different allowable probabilities of failure. General relations are derived between probability of failure and weight of individual members for a variety of conditions of loading. Types of failure considered include buckling, creep, etc. As an illustration, for mathematical simplicity, the paper assumes Gaussian distribution for a number of parameters and take the corresponding coefficients of variation as constant. (The latter assumption is perhaps more reasonable than the former.) From here it derives a number of design charts and comparative diagrams. It is concluded that, given an over-all probability of failure, the proposed approach effects economies in weight of 2 to 8 percent, under usual conditions, in comparison with conventional methods using constant factor of safety.

51. Sandorff, P.E.
Structures considerations in design for space boosters. J. AM. ROCKET SOC. v. 30, n. 11, p. 999-1008, Nov 1960.

51. (cont'd) A description of the ways in which structures problems influence and are influenced by the other aspects of early space vehicle design. The nature of the major loading conditions and how they may vary with vehicle size, configuration and design parameters is discussed. Structural design methods available to support the major vehicle loads are compared, using structural indexes to assess future design problems. Functional components of the structure are analyzed to determine how weight may vary with vehicle design parameters. Dead weight data available on existent designs are summarized, and the trends noted. In support of the predictions derived analytically, a family of large booster vehicle designs is presented.

52. Hu, T.C. and Shield, R.T.
Uniqueness in the optimum design of structures.
TRANS. ASME, J. APPL. MECH. E, v. 83,
p. 284-287, 1961.

A procedure was developed previously by Shield for the determination of the optimum design of a sandwich shell or structure. The uniqueness of the optimum design obtained by this procedure is investigated and it is shown that all optimum designs admit a common collapse mode. This result is used to prove the uniqueness of minimum weight designs obtained in previous work.

53. Barnett, R. L. and Humphreys, A.
Design for minimum weight. MATERIALS
IN DESIGN ENG. v. 55, p. 83-85, Jan 1962.

Charts giving merit index of some 125 metallic and nonmetallic materials upon the basis of (1) specific stiffness, (2) weight/strength ratio, (3) specific compressive strength, and (4) specific tenacity are presented. By the use of the charts, the weight penalty which must be paid in order to obtain a desired strength or stiffness can be calculated.

54. Kalaba, R.
Design of minimal weight structures for
given reliability and cost. J. AEROSPACE
SCI. v. 29, p. 355, 356, Mar 1962.

Procedure for the design of minimal-weight mechanical structures having a given reliability, to provide an improved computational procedure and include the cost of materials.

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